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# Longitudinal and spin Hall conductance of a one-dimensional Aharonov–Bohm ring

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**Abstract**
The longitudinal and spin Hall conductances of an electron gas with Rashba–Dresselhaus spin–orbit interaction, confined to a quasi-one-dimensional Aharonov–Bohm ring, are studied as functions of disorder and magnetic flux. The system is mapped onto a one-dimensional virtual lattice and is described, in a tight binding approximation, by a Hamiltonian that depends parametrically on the nearest neighbour hopping integral $t$, the Rashba spin–orbit coupling $V_R$, the Dresselhaus spin–orbit coupling $V_D$ and an Anderson-like, on-site disorder energy strength $W$. Numerical results are obtained within a spin dependent Landauer–Büttiker formalism.

(Some figures in this article are in colour only in the electronic version)

## 1. Introduction

One of the most exciting developments that has resulted from the analysis of spin transport in ideal two-dimensional (2D) structures is the theoretical prediction of dissipationless spin currents induced by electric fields. The flow of spin-polarized carriers in a direction perpendicular both to the driving electric field and to the spin polarization, that occurs in the presence of a spin–orbit (SO) interaction, is equivalent to an intrinsic spin Hall effect. In the n-doped semiconductors, the spin–orbit coupling that determines this behaviour originates either from the inversion asymmetry of the confining potential in the direction perpendicular to the electron system (Rashba [1]) or from the bulk asymmetry (Dresselhaus [2]). In the case of the Rashba coupling, the calculated spin Hall conductivity has been found to be equal to a constant, universal value [3], $\sigma_{sH} = e/8\pi$.

The physical existence, i.e. the experimental detection, of the spin Hall effect and its robustness in disordered systems have been subjects of considerable theoretical debate. It has been shown by a number of authors that for a 2D electron system with pure Rashba or Dresselhaus coupling, the spin Hall conductance becomes zero when impurity scattering is considered [4]. More recently, this conclusion has been extended also to 2D holes in III–V materials [5]. These results buttress the observation that pure spin currents occur even in the absence of the driving fields and not associated with the actual motion of spins [6]. The opposite view argues that in hole-doped semiconductors, the vertex corrections to the conductivity resulting from impurity scattering are zero, and consequently the spin Hall effect persists [7]. The extension of these ideas to mesoscopic ring structures has also been actively pursued. The circular geometry favours quantum interference effects resulting from the phase difference acquired by electrons. Therefore, an oscillatory behaviour of the spin Hall conductance is obtained. New phenomena, such as the detection of a pure spin current in the transverse voltage probes, when conventional unpolarized current is injected through the longitudinal leads of a ring, are being predicted [8]. Experimental measurements of spin resolved currents in *finite* size 2D systems show the possible existence of the effect [9, 10], as predicted theoretically from numerical simulations [7]. A definite experimental resolution of this controversy is still awaiting discovery.

In this paper we develop a unitary formalism whose consistent application leads to numerical values of the longitudinal, $G_L$, and spin Hall, $G_s$, conductances of an electron gas with spin–orbit coupling confined to a quasi-one-dimensional Aharonov–Bohm ring as functions of disorder and magnetic flux. The dependence of these results on the ratio of the strengths of the two SO interactions is also shown. We start by mapping the Hamiltonian of the system to a one-dimensional virtual lattice in a tight binding approximation. This approach allows the simultaneous incorporation of both the Rashba–Dresselhaus terms of the spin–orbit interaction, as amplitudes of hopping between nearest neighbour sites of the virtual lattice, and that of disorder as an on-site potential, like in the Anderson model. The exact eigenfunctions of this Hamiltonian are used to calculate the conductances via the Landauer–Büttiker formula, where the transmission matrix elements are estimated with real space Green's functions.

## 2. Theoretical framework

### *2.1. The Hamiltonian*

The motion of an electron in the quasi-one-dimensional Aharonov–Bohm ring is described by

$$H = \frac{\hbar^2}{2m^\star}\left(-\mathrm{i}\partial_\varphi + \frac{\phi}{\phi_0}\right)^2 + \frac{1}{r}[(\alpha\cos\varphi + \beta\sin\varphi)\sigma_x + (\alpha\sin\varphi + \beta\cos\varphi)\sigma_y] \times \left(-\mathrm{i}\partial_\varphi + \frac{\phi}{\phi_0}\right) - \frac{\mathrm{i}}{2r}[(\alpha\cos\varphi + \beta\sin\varphi)\sigma_y + (\alpha\sin\varphi + \beta\cos\varphi)\sigma_x], \tag{1}$$

where $\mathbf{p} = \mathbf{p}_0 - (e/c)\mathbf{A}$ is the generalized electron momentum in the magnetic field, $\sigma_i$ are the Pauli matrices, while $\alpha$ ($\beta$) is the Rashba (Dresselhaus) spin–orbit coupling strength. We adopt the standard geometry where $\mathbf{A}$ describes a magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$ along the $z$ direction, $\mathbf{B} = (0, 0, B)$. The corresponding magnetic flux $\phi = \pi r^2 B$ is a tunable parameter needed to study the magnetoconductance in the Aharonov–Bohm ring and $\phi_0$ is the magnetic flux quantum $\phi_0 = hc/e$.

This is the Hamiltonian obtained in [11] generalized to incorporate the Dresselhaus term of the SO coupling, proportional to $\beta$, in addition to the usual Rashba term, proportional to $\alpha$. The additional parameter introduced by the geometry of the system is $\varphi$, the angular coordinate. The relative strength of the Rashba and Dresselhaus terms, $\alpha/\beta$, describing the spin–orbit coupling in semiconductor quantum wells, is available from photocurrent measurements [12]. The interplay of the two SO couplings has also been lately subject to intensive theoretical investigation [13, 14] as regards other physical phenomena in 2DEG, quantum wells and quantum dots.

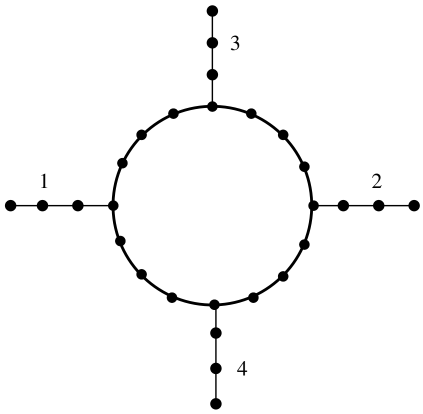


**Figure 1.** Graphical depiction of the lattice model used for computing the longitudinal and spin Hall conductance in a one-dimensional Aharonov–Bohm ring. Four metallic leads are attached to the ring, acting as: injector (1), detector (2) and voltage probes (3 and 4).

In the absence of the magnetic field, $\phi = 0$, the 1D spinor that diagonalizes equation (1) has a general form given by

$$\phi^s_{\lambda,n}\varphi = \mathrm{e}^{\mathrm{i}\lambda n\varphi}\begin{pmatrix}\chi_1\\ \chi_2\mathrm{e}^{\mathrm{i}\varphi}\end{pmatrix}, \tag{2}$$

with $\lambda = \pm 1$ (travel direction), $n \geqslant 0$ (the orbital quantum number) and $s = \pm 1$ (spin), and the corresponding eigenvalues are $E_{\lambda,n,s}$. An exact analytic solution in the case of a pure Rashba coupling is discussed in detail in [15]. Its generalization to include also the Dresselhaus term is discussed elsewhere [16].

For finite values of the magnetic flux, the problem becomes much more complicated and only numerical solutions are possible. To resolve this difficulty, we map our system onto a virtual lattice and consider a Hamiltonian, in a tight binding approximation, of the form

$$H_{\mathrm{ring}} = \sum_{n=1}^{N}\varepsilon_n c_n^\dagger c_n - \sum_{n=1}^{N}(t_{n,n+1}c_n^\dagger c_{n+1} + \mathrm{h.c.}). \tag{3}$$

Here $c_j^\dagger$ is the creation operator at site $j$ for a spinor $c_j^\dagger = (c_{j,\uparrow}^\dagger c_{j,\downarrow}^\dagger)$. The first term in equation (3) is the on-site disorder, as in the Anderson model, with a random energy generated by a box distribution $[-W/2, W/2]$. The on-site term also contains components of the non-derivative contributions from the SO interaction term from equation (1). This contribution is off-diagonal in the spin index and diagonal only in the site index. The derivative components of the SO interactions are incorporated in the hopping term which acquires position and spin dependence. The prototype structure for our calculation is presented in figure 1. The metallic leads attached to the sample are considered perfect semi-infinite wires, without disorder and spin–orbit coupling.

If $\delta$ is the virtual lattice constant in the ring, the hopping couplings for the SO interaction are, respectively, $t_0^{(\mathrm{R})} = \alpha/2\delta$ for Rashba and $t_0^{(\mathrm{D})} = \beta/2\delta$ for Dresselhaus, leading to phase dependent couplings equal to

$$\begin{aligned}
t^{(0)}_{n,n+1} &= t_0\mathrm{e}^{\mathrm{i}(2\pi/N)(\phi/\phi_0)},\\
t^{(\mathrm{R})}_{n,n+1} &= -\mathrm{i}t_0^{(\mathrm{R})}\left[\cos\frac{\varphi_n+\varphi_{n+1}}{2}\sigma_x + \sin\frac{\varphi_n+\varphi_{n+1}}{2}\sigma_y\right]\\
t^{(\mathrm{D})}_{n,n+1} &= -\mathrm{i}t_0^{(\mathrm{D})}\left[\sin\frac{\varphi_n+\varphi_{n+1}}{2}\sigma_x + \cos\frac{\varphi_n+\varphi_{n+1}}{2}\sigma_y\right].
\end{aligned} \tag{4}$$

If we consider $\delta = 5.0$ nm the lattice constant, then for number of sites in the ring $N = 250$, and for $m^\star = 0.068m$ the effective mass of the electron (in GaAs), the hopping integral

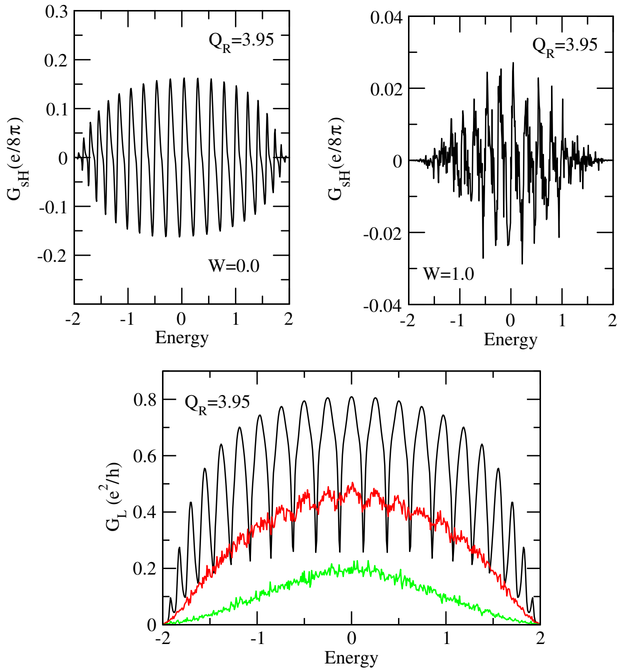


**Figure 2.** Upper panel: spin Hall conductance as a function of the Fermi energy for a 1D Aharonov–Bohm ring for $Q_R = 3.95$, $Q_D = 0.0$. The disorder strength is $W = 0$ (left) and $W = 1.0$ (right). Lower panel: longitudinal conductance as a function of the Fermi energy, for different disorder strengths $W = 0.0$, $W = 1.0$ and $W = 2.0$. The number of sites in the ring is 100.

is $t \simeq 21.6$ meV, and the radius of the ring is $r \simeq 0.2\ \mu$m. A typical value for the Rashba coupling [17] is below 80 meV Å, which corresponds to $t_0^{(R)} \simeq 1.6$ meV. A general adiabaticity condition was proposed in [18] $Q \gg \sqrt{N}$, where $N$ is the number of scattering events. For diffusive one-dimensional rings this condition is similar to $Q \gg L/l$, where $L$ is the characteristic length scale of the system over which the effective field due to spin–orbit interaction changes and $l$ is the mean free path. We followed [15] and introduced the adiabaticity parameters $Q_{R(D)}$ as $Q_{R,(D)} = 2t_0^{(R),(D)}/t_0(N/\pi)$. Typical values for $Q_{R,(D)}$ considered in this analysis are $Q_{R,(D)} \in [0, 10]$.

In the LB theory, the total current in terminal $p$ is given by the addition of the transmission probabilities of electrons from all the other leads: $I_p = e^2/h \sum_{q\neq p} T_{pq}(V_p - V_q)$ (the sum is over all the other leads $q$ connected to the system). The spin current is similarly defined to be $I_{p,\mu}^{\text{spin}} = e/(4\pi) \sum_{q\neq p,\nu} T_{pq}^{\mu\nu}(V_p - V_q)$ (here $\mu$ and $\nu$ stand for the spin indices). Ballistic transport between the leads is assumed. $T_{pq}^{\mu\nu}$ represents the transmission probability over all the conduction channels, for detecting a spin $\mu$ in the lead $p$ arising from an injected spin $\nu$ electron in lead $q$, when both spin-flip and non-spin-flip processes are considered.

The total scattering between two leads $p$ and $q$ can be simply written as the sum over all spin components $T_{pq} = T_{pq}^{\uparrow\uparrow} + T_{pq}^{\uparrow\downarrow} + T_{pq}^{\downarrow\uparrow} + T_{pq}^{\downarrow\downarrow}$. Two other useful combinations [19] are $T_{pq}^{\text{in}} = T_{pq}^{\uparrow\uparrow} + T_{pq}^{\uparrow\downarrow} - T_{pq}^{\downarrow\uparrow} - T_{pq}^{\downarrow\downarrow}$ and $T_{pq}^{\text{out}} = T_{pq}^{\uparrow\uparrow} + T_{pq}^{\downarrow\uparrow} - T_{pq}^{\uparrow\downarrow} - T_{pq}^{\downarrow\downarrow}$. $T_{pq}^{\text{out}}$ represents the difference between the transmission probabilities for detecting an electron in the lead $p$ arising from an injected spin $\uparrow$ ($\downarrow$) electron in lead $q$. The spin Hall conductance is then computed

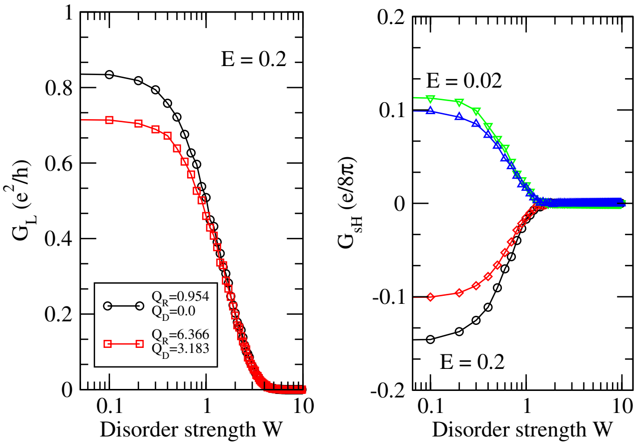


**Figure 3.** Longitudinal conductance (left) and spin Hall conductance (right) as functions of the disorder for different spin–orbit coupling strengths. In the right panel $Q_R = 0.954$, $Q_D = 0.0$ (○ and ▽) and $Q_R = 6.366$, $Q_D = 3.183$ (◇ and △). The results are for two Fermi energy values ($E_F = 0.2$ and 0.02). The numbers of sites in the ring are 100, and the average is over 1000 different configurations.

according to

$$G_{sH} = (I_{3,\uparrow}^{\text{spin}} - I_{3,\downarrow}^{\text{spin}})/V_1, \tag{5}$$

while the longitudinal conductance

$$G_L = I_2/(V_1 - V_2). \tag{6}$$

In our geometry, terminals 3 and 4 are voltage probes and the current flows between terminals 1 and 2. By using the expressions for the voltages obtained by inverting the multiprobe equations the spin Hall conductance is given by $G_{sH} = e/(8\pi)(T_{13}^{\text{out}} + T_{43}^{\text{out}} + T_{23}^{\text{out}} - T_{34}^{\text{in}} - 2T_{31}^{\text{in}})$, and the expression for longitudinal conductance is $G_L = e^2/h(T_{21} + 0.5\,T_{32} + 0.5\,T_{42})$.

The zero-temperature conductance, **G**, is directly related to the transmission matrix **T**, which is a matrix in spin indices when spin–orbit coupling is present in the system. The transmission matrix $T_{pq}^{\alpha\beta}$ describes the transmission probability over all the conduction channels for detecting a spin $\alpha$ in the lead $p$ arising from the spin-flip processes, or lack of them, of an injected spin $\beta$ electron in lead $q$. The retarded Green's function $G_R = (E_F - H - \sum_{p=1}^{4} \Sigma_p)^{-1}$ ($E_F$ is the Fermi energy and $H$ is the Hamiltonian for the sample given by equation (3)) and its Hermite conjugate, the advanced Green's function ($G_A = G_R^{\dagger}$), allow the calculation of the transmission matrix: $T_{pq}^{\alpha\beta} = \text{Tr}[\Gamma_p^{\alpha} G_R \Gamma_q^{\beta} G_A]$. Here $\Gamma_p^{\alpha} = \text{i}(\Sigma_p^{\alpha} - \Sigma_p^{\alpha\dagger})$ with $\Sigma_p^{\alpha}$ the retarded self-energy, entirely related to the lead–sample scattering. In our analysis we use the tight binding approximation, extensively discussed in [20], for calculating the self-energy. When each terminal is modelled by a semi-infinite perfect wire the self-energy matrix is diagonal in spin indices $\Sigma_p^{\uparrow} = \Sigma_p^{\downarrow}$.

### *2.2. Results and discussion*

Within the framework outlined above, we obtain numerical estimates for the longitudinal and spin Hall conductances of a four-terminal ring in the presence of a magnetic field. Both conductances are studied as functions of the Fermi energy ($E_F$), disorder strength ($W$), dimensionless magnetic flux ($\phi/\phi_0$) and adiabaticity parameters $Q_{R(D)}$. In figure 2 we present

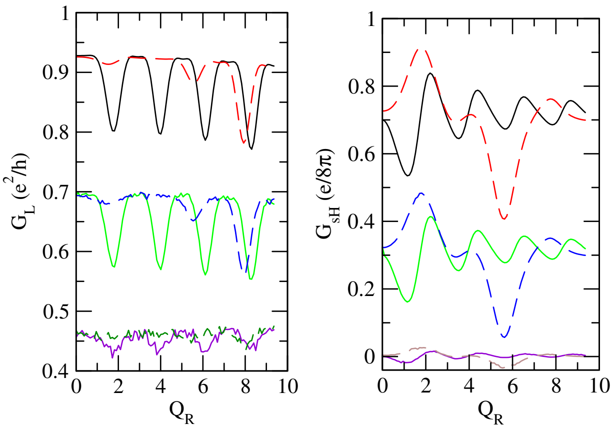


**Figure 4.** Longitudinal and spin Hall conductance as a function of the Rashba spin–orbit strength for two different Dresselhaus couplings and for different disorder strengths. Solid lines represent results for $Q_D = 0.0$ and dashed lines correspond to $Q_D = 3.183$. The values of $W$ are 0, 0.3 and 0.5. Left: results for $W = 0$ were shifted up by 0.2 for clarity. Right: results for $W = 0$ were shifted by 0.7 and for $W = 0.3$ by 0.3. The system size is 100 sites and the average for disordered samples was taken over 1000 samples. In both figures, the Fermi energy is $E_F = 0.2$.

results for $G_L$ and $G_{sH}$ as functions of the Fermi energy. The upper left panel presents results for a ballistic system, when no disorder is present, while the right panel presents resulting values of disorder big enough to destroy the spin Hall effect. In the lower panel we show results for the longitudinal conductance for different disorder strengths. The strong oscillations in $G_L$ and $G_{sH}$ are due to the discrete nature of the energy spectrum in the system. In figure 3 we present the effect of disorder for a fixed value of the Fermi energy. For $W > 1$ the spin Hall effect is practically absent and a vanishing value for $G_{sH}$ is obtained. It is well known by now that the longitudinal conductance of an Aharonov–Bohm ring with a Rashba SO interaction coupled to two external terminal is oscillating in nature. This effect was demonstrated theoretically, both by using analytical [21] and by using numerical [15] methods. The effect of SO scattering in disordered mesoscopic systems was also studied. It was shown [22] that for some values of SO strength the transport quantities that are periodic exhibit halving of the period. We did numerical analysis to confirm this behaviour by computing the harmonics of the longitudinal conductance for a two-contact system (results not shown here), when the longitudinal conductance was obtained from ensemble averaging over independent disorder configurations.

In figure 4 we present the effect of spin–orbit coupling, of both Rashba and Dresselhaus types, on $G_L$ and $G_{sH}$ when the ring is connected to four terminals. Oscillating behaviour is observed, similarly to in the case of a two-terminal structure. We observe that, on increasing the disorder, the longitudinal conductance's periodicity is preserved, but the amplitude of the oscillations is strongly reduced. For $G_{sH}$, damping oscillating behaviour is observed. An analytic calculation, shown elsewhere [16], indicates that the modulation damping function is proportional to $1/\sqrt{(1 + Q_R^2)}$ for a system with pure Rashba spin–orbit coupling and in the absence of an external magnetic field. The Dresselhaus SO term has a strong effect on both $G_L$ and $G_{sH}$. First, the periodicity as a function of $Q_R$ is modified and, secondly, the amplitude of the oscillations is strongly reduced in the limit $Q_R < Q_D$. In our geometry, $G_{sH} = 0$ when $Q_R = Q_D$ for any disorder strength, similarly to in the case of a square lattice [16]. In

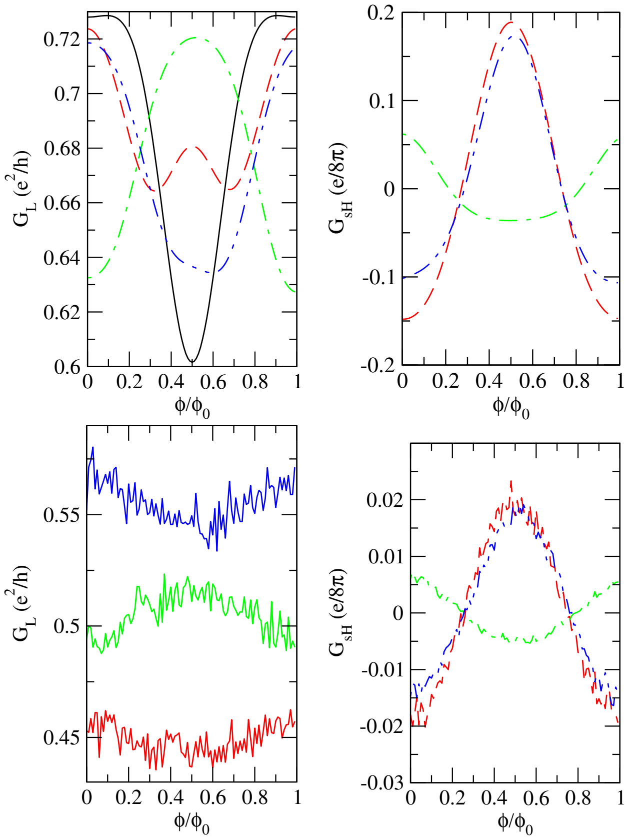


**Figure 5.** (Upper panel) Left: longitudinal conductance as a function of magnetic flux for different spin–orbit coupling strengths. ($Q_R = 0.0$, $Q_D = 0.0$ (solid line), $Q_R = 0.954$, $Q_D = 0.0$ (dashed line), $Q_R = 6.366$, $Q_D = 0.0$ (dash–dotted line) and $Q_R = 6.366$, $Q_D = 3.183$ (dash–dot–dotted line).) Right: spin Hall conductance as a function of magnetic field, with similar parameters and the same line-styles. (Lower panel) Left: effect of disorder on the longitudinal conductance, for $Q_R = 0.954$, $Q_D = 0.0$ (red line), $Q_R = 6.366$, $Q_D = 0.0$ (green line) (data are shifted by +0.05) and $Q_R = 6.366$, $Q_D = 3.183$) (blue line) (data are shifted by 0.1 for clarity). Right: spin Hall conductance in the presence of disorder for the same parameters as in the upper right panel. The overall behaviour is preserved but the amplitude is strongly reduced in the presence of disorder. The disorder strength is $W = 1.0$. All figures: the number of sites is 100 and the Fermi energy was fixed to $E_F = 0.2$.

figure 5 we present the effect of an external magnetic field. The interference effects in the conductivity, as a function of the magnetic flux, predicted by Al'tshuler *et al* [23] originate from the degeneracies of several eigenstates in the ordered system [24]. We present results for the clean limit and for disorder strengths which practically destroy the spin Hall effect. In a ring attached to four terminals, similar oscillations are observed in the longitudinal conductance.

## 3. Conclusions

In the framework of the tight binding formalism we have computed the spin Hall conductance and the longitudinal conductance for a one-dimensional Aharonov–Bohm ring attached to four external terminals.

Our results strengthen the notion of the possibility of obtaining a spin-polarized current in the transverse direction when a conventional unpolarized current is injected in the longitudinal direction. We studied the effect of Rashba and Dresselhaus spin–orbit coupling, disorder and external magnetic field on transport properties. Our calculations are based on the Landauer–Büttiker formalism, combined with a Green's function approach. Both $G_L$ and $G_{sH}$ are oscillating as functions of the Fermi energy both for clean samples and when disorder is present, due to the discrete nature of energy spectrum for a finite ring. The critical disorder that destroys the spin Hall effect was found to be close to $W_C \simeq 1$ in our model. When an external magnetic field threatens the AB ring, interference effects are presents as functions of the magnetic flux, as predicted by Al'tshuler [23].

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